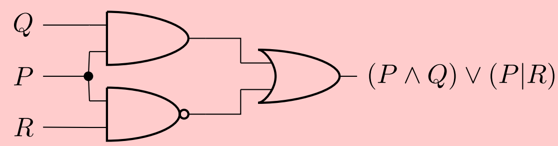
Spring 2016 Math 245 Mini Midterm 1 Solutions

1. Draw a circuit corresponding to $(P \wedge Q) \vee (P|R)$. Label carefully.



2. Fill in the blanks to prove the following theorem: $((\neg p \vee r) \wedge (t \to q) \wedge (q \to p)) \to r$

	proposition	justification
1.	$(\neg p) \vee r$	hypothesis
2.	$t \to q$	hypothesis
3.	$q \to p$	hypothesis
4.	t	tautology is always true
5.	q	modus ponens on 2,4
6.	p	modus ponens on 3,5
7.	$\therefore r$	disjunctive syllogism on 1,6

3. Carefully define each of the following terms:

a. contradiction

A **contradiction** is a (typically compound) proposition that is always true.

b. vacuously true

A conditional proposition $p \to q$ is **vacuously true** if p is false (regardless of q).

c. converse

The **converse** of conditional proposition $p \to q$ is the conditional proposition $q \to p$.

d. disjunctive addition

The rule of inference **disjunctive addition** allows us to conclude $p \vee q$ from the hypothesis p.

e. predicate

A **predicate** is a collection of propositions, indexed by one or more variables, each drawn from some domain of discourse.

4. Write and simplify the negation of the proposition:

$$\forall x \in \mathbb{R}, \text{ if } x(x+1) > 0 \text{ then } x > 0 \text{ or } x < -1.$$

$$\exists x \in \mathbb{R} \text{ with } x(x+1) > 0 \text{ and } -1 \le x \le 0$$

5. Prove that the conditional proposition $p \to q$ is equivalent to its contrapositive.

Method 1: $(p \to q) \equiv^1 (q \vee (\neg p)) \equiv^2 ((\neg\neg q) \vee (\neg p)) \equiv^3 ((\neg p) \vee (\neg\neg q)) \equiv^1 ((\neg q) \to (\neg p))$

1: Theorem on conditionals 2: Theorem on double negations 3: Theorem on disjunctions

Method 2: Truth table:

p	q	$p \to q$	$\neg p$	$\neg q$	$(\neg q) \to (\neg p)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

The third and sixth columns agree, so the theorem is proved.